HOMEWORK 1 - ANSWERS TO MOST PROBLEMS

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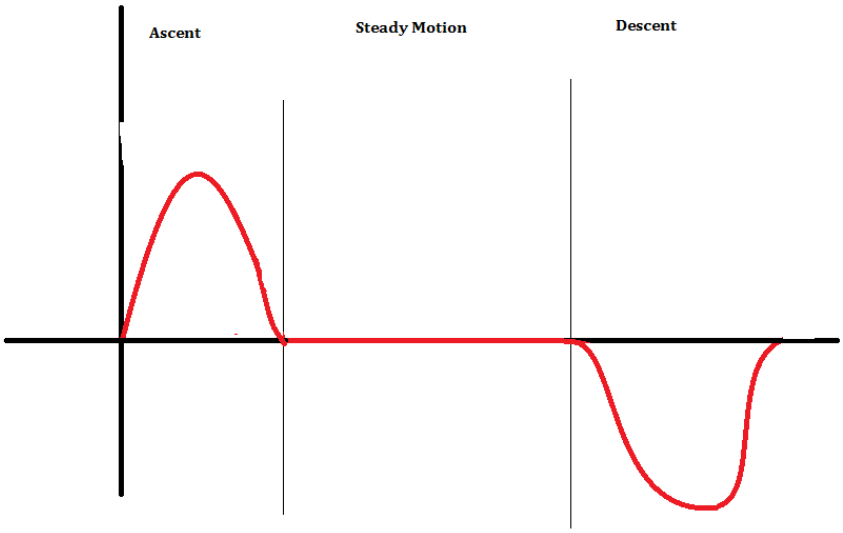
1. Section 1.1: Four ways to represent functions

1.1.6. Yes (by the vertical line test), Domain $= [-2, 2]$, Range $= [-1, 2]$

1.1.18.

(a) The graph of $x(t)$ should just be a line going through the origin
(b) The graph of $y(t)$ should look at first like the right half of a parabola, then should be constant for a while, and then look like the left half of a parabola
(c) The graph of the horizontal velocity looks like a horizontal line
(d) See announcement on bspace for a detailed solution! The picture you get is:

1A/Solutions/Vertical Velocity.png



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1.1.32. Domain $= [-2, 2]$, Range $= [0, 2]$, Graph is just the upper-half of the circle centered at 0 of radius 2.

1.1.45. $f(x) = \frac{5}{2}x - \frac{11}{2}$

1.1.57. $V(x) = x(20 - 2x)(12 - 2x)$ (no need to expand the answer!)

1.1.61. f is odd, g is even

2. Section 1.2: Mathematical models: a catalog of essential functions

1.2.2.

(a) Rational function
(b) Algebraic function
(c) Exponential function
(d) Power function
(e) Polynomial of degree 6
(f) Trigonometric function

1.2.4.

(a) G
(b) f
(c) F
(d) g

1.2.8. (a) $y = 2(x - 3)^2$, (b) $y = -x^2 - \frac{5}{2}x + 1$

1.2.16.

(a) $C(x) = 13x + 900$ (C is the cost and x is the number of chairs produced)
(b) 13; Cost per chair
(c) 900; Start-up cost (i.e. money needed to buy machines in order to *start* producing chairs)

3. Section 1.3: New functions from old functions

1.3.1.

(a) $y = f(x) + 3$
(b) $y = f(x) - 3$
(c) $y = f(x - 3)$
(d) $y = f(x + 3)$
(e) $y = -f(x)$
(f) $y = f(-x)$
(g) $y = 3f(x)$
(h) $y = \frac{1}{3}f(x)$

1.3.7. $y = -\sqrt{3(x + 4) - (x + 4)^2} - 1$

1.3.14. Basically compress the graph of $\sin(x)$ horizontally by a factor of 3 (notice that the new period now is $\frac{2\pi}{3}$ and then stretch the resulting graph vertically by a factor of 4 (so the new graph has range $[-4, 4]$ instead of $[-1, 1]$)

1.3.30.

(a) $(f+g)(x) = \sqrt{3-x} + \sqrt{x^2-1}$
(b) $(f-g)(x) = \sqrt{3-x} + \sqrt{x^2-1}$
(c) $(fg)(x) = \sqrt{3-x} \times \sqrt{x^2-1}$
(d) $\left(\frac{f}{g}\right)(x) = \frac{\sqrt{3-x}}{\sqrt{x^2-1}}$

All of those functions have domain $(-\infty, -1] \cup [1, 3]$ **EXCEPT** for (d), which has domain $(-\infty, -1) \cup (1, 3]$

1.3.36.

(a) $(f \circ g)(x) = \frac{\sin(2x)}{1+\sin(2x)}$; Dom = all odd multiples of $\frac{\pi}{2}$
(b) $(g \circ f)(x) = \sin\left(\frac{2x}{1+x}\right)$; Dom = all real numbers except -1
(c) $(f \circ f)(x) = \frac{\frac{x}{1+x}}{1+\frac{x}{1+x}} = \frac{x}{1+2x}$; Dom = all real numbers except $\frac{-1}{2}$ and -1
(d) $(g \circ g)(x) = \sin(2\sin(2x))$; Dom = all real numbers

4. Section 1.4: Graphing Calculators and Computers

Don't worry about this section, it's not very important and it won't be on the exam!

5. Section 1.5: Exponential Functions

1.5.3. Basically, the larger the base, the faster the function is increasing

1.5.5. Notice that $\left(\frac{1}{3}\right)^x = 3^{-x}$, which means that $\left(\frac{1}{3}\right)^x$ is the reflection of 3^x across the y-axis! Similarly with 10^x.

1.5.6. The smaller the base, the faster the function is going to 0.

1.5.16. (a) All real numbers ; (b) All ≤ 0 real numbers

1.5.17. $f(x) = 3 \cdot 2^x$

1.5.18. $f(x) = 2 \cdot \left(\frac{1}{3}\right)^x = 2 \cdot 3^{-x}$